

# Verification of Timed Systems

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# The satisfaction problem

Given a system  $S$  and a property  $\varphi$  show that

$$S \models \varphi$$

The choice of which **logical language** to use for writing  $\varphi$  depends on the underlying computational paradigm

# A logical language for timed systems

Variant of **Computation Tree Logic** with two types of formulae

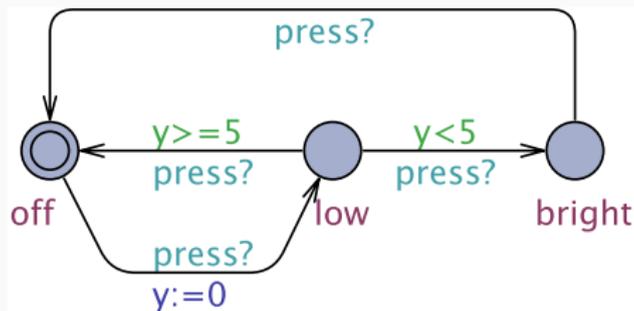
description of **state** and **path** properties

## Grammar

$$\Psi ::= \ell \mid c \mid \text{deadlock} \mid \text{not } \Psi \mid \Psi \text{ and } \Psi$$

We can thus express **current locations**  $\ell$ , **clock constraints**  $c \in \mathcal{C}(C)$ , and the presence of **deadlocks**

# Back to the annoying lamp



## Exercise

Write formulae for the following statements

1. The lamp is on `low` mode
2. Not `off` and  $y > 25$
3. If it is `low` or `bright` then  $y \leq 3600$

## Grammar

$$\Pi ::= A \square \Psi \mid A \diamond \Psi \mid E \square \Psi \mid E \diamond \Psi \mid \Phi \rightsquigarrow \Psi$$

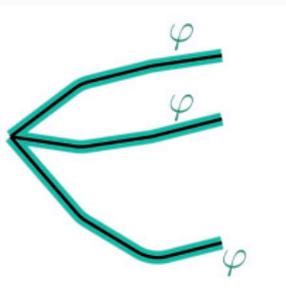
$A, E$  quantify (universally / existentially) over paths

$\square, \diamond$  quantify (universally / existentially) over states in a path

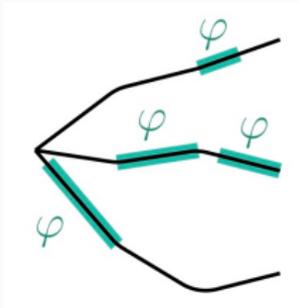
Paths are seen as possible system executions

# Path Formulae in Pictures

$A \Box \varphi$



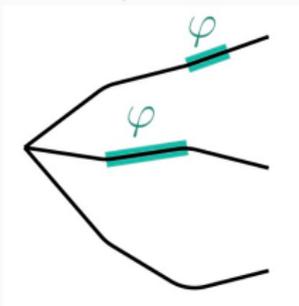
$A \Diamond \varphi$



$E \Box \varphi$



$E \Diamond \varphi$



# Safety properties

$A \Box \varphi$     $E \Box \varphi$

Something bad will never happen, e.g.

- Temperature will never exceed the prescribed threshold
- System never reaches a deadlock
- At least one execution with no deadlocks

$E \diamond \varphi$

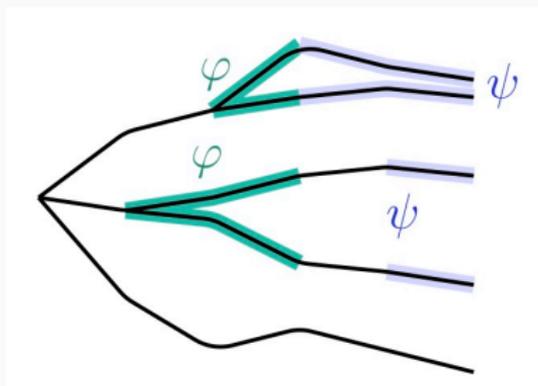
Something good can happen, e.g.

- All adventurers reach the other side
- All adventurers reach the other side in  $\leq 17$  minutes

# Path formulae

For all paths if  $\varphi$  holds at some point then  $\psi$  will also hold later on

$$\phi \rightsquigarrow \psi \stackrel{\text{abv}}{=} A\Box(\phi \rightarrow A\Diamond\psi)$$



# Liveness Properties

$A \diamond \phi \quad \phi \rightsquigarrow \psi$

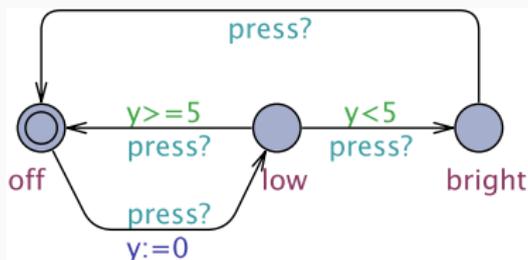
If something happens then something good eventually happens

- When pressing ON the television will eventually turn on
- If the philosopher requests a fork she will get it
- If the plane asks to land it will eventually land

Write the sentences below in CTL

1. The system never enters in deadlock
2. The location  $\ell$  is reachable
3. In all executions we reach location  $\ell$
4. If we reach location  $\ell$  we will inevitably reach location  $s$
5. There exists at least one execution where variable  $i$  is always below or equal 10
6. The two philosophers never eat at the same time

# Back to the Annoying Lamp



## Exercise

1. The lamp can become bright;
2. The lamp will eventually become bright;
3. The lamp can never be on for more than 3600s;
4. It is possible to never turn on the lamp;
5. Whenever the light is bright, the clock  $y$  is non-zero;
6. Whenever the light is bright it will eventually become off.

## Chapter's Conclusion

Communicating systems as **timed automata**



Syntax

**Semantics** for rigorous analysis and verification

**UPPAAL** as an important tool of the cyber-physical engineer

# Key takeaways

**Time** as the main physical process ...

... others will appear in the next lectures

Barely scratched the surface ...

... more about the theory of timed automata in the website