

Verification of Timed Systems

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The Satisfaction Problem

Given a system S and a property φ show that



The choice of which **logical language** to use for writing φ depends on the underlying computational paradigm

Variant of Computation Tree Logic with two types of formulae

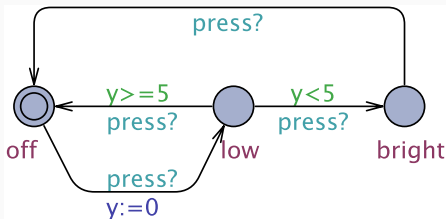
description of **state** and **path** properties

Grammar

$$\Psi ::= \ell \mid c \mid \text{deadlock} \mid \text{not } \Psi \mid \Psi \text{ or } \Psi \mid \Psi \text{ and } \Psi$$

We can thus express **current locations** ℓ , **clock constraints** $c \in \mathcal{C}(C)$, and the presence of **deadlocks**

Back to the lamp



Exercise

Write formulae for the following statements

1. The lamp is low
2. Not off and $y > 25$
3. If it is low or bright then $y \leq 3600$

Grammar

$$\Pi ::= A \square \Psi \mid A \diamond \Psi \mid E \square \Psi \mid E \diamond \Psi \mid \Phi \rightsquigarrow \Psi$$

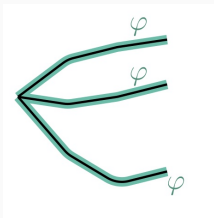
where

- A, E quantify (universally and existentially, resp.) over **paths**
- \square, \diamond quantify (universally and existentially, resp.) over **states in a path**

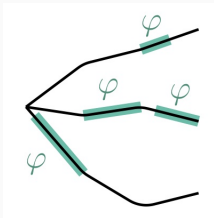
Paths can be seen as possible executions

Path Formulae pt. II

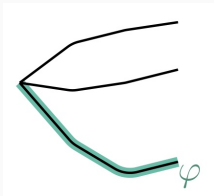
$A\Box\varphi$



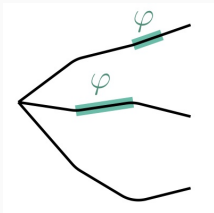
$A\Diamond\varphi$



$E\Box\varphi$



$E\Diamond\varphi$



$A \Box \varphi$ and $E \Box \varphi$

Something bad will **never** happen

Examples:

- A nuclear reactor's temperature will never exceed a certain threshold
- We will never reach deadlock
- There is at least one execution in which we never reach deadlock

Reachability Properties

$E\Diamond\varphi$

Something good **can** happen

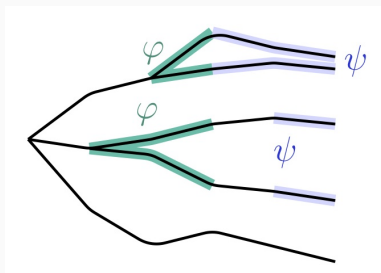
Examples:

- All adventurers reach the other side.
- All adventurers reach the other side in ≤ 17 minutes.
- ...

Path Formulae pt. III

For all paths if φ holds at some point then ψ will also hold later on

$$\varphi \rightsquigarrow \psi$$



Liveness Properties

$A \diamond \phi$ and $\phi \rightsquigarrow \psi$

Something **good** will **eventually** happen or
if something happens then something **good** will **eventually** happen

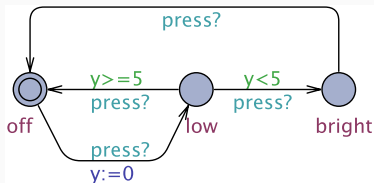
Examples:

- **Always** when pressing the on button the television will eventually turn on
- If the philosopher requests a fork she will **eventually** get it
- If the plane asks to land it will **eventually** land

Write the sentences below in CTL

1. The system never enters in deadlock
2. The location ℓ is reachable
3. In all executions we reach location ℓ
4. If we reach location ℓ we will inevitably reach location s
5. There exists at least one execution where variable i is always below or equal 10
6. The two philosophers never eat at the same time

Back to the lamp



Exercise

1. The lamp can become bright;
2. The lamp will eventually become bright;
3. The lamp can never be on for more than 3600s;
4. It is possible to never turn on the lamp;
5. Whenever the light is bright, the clock y is non-zero;
6. Whenever the light is bright it will eventually become off.