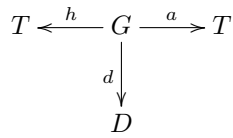## MFES/CSI/2019-11-07 — Specifying a football championship

The teams (T) of a football league play games (G) at home or away, and every game takes place in some date:

$$T \xleftarrow{\ h\ } G \xrightarrow{\ a\ } T$$
$$d \downarrow$$
$$D$$

Desirable properties:

$a \cdot h^\circ = \top - id$ -- teams play all against each other but don't play with themselves
$id \leqslant \langle a, h \rangle$ -- no repeated games with the same teams
$id \leqslant \langle a \cup h, d \rangle$ -- no team playing twice on same day

Observations:

- The relation $\frac{\langle a,h \rangle}{\langle h,a \rangle}$ relates a game with its "symmetric" – that is, the one with home and away teams swapped.
- This relations is *symmetric*, as expected. (Check this.)
- It should be a function too. Because of its symmetry, it will be an isomorphism (bijection) on G.
- Final requirement: s is a function.

By running this model in Alloy will "solve" the problem of finding a schedule for a number of n teams playing $n\,(n-1)$ games.

```
    -- (c) MFES / CSI

    -- Games

    sig G{
        a : one T,
        h : one T,
        d : one D,
        s : one G
    }

    -- Teams and dates

    sig D,T {}

    fact{

        (T->T) - iden = ~h.a          -- teams play all against each other
                                      -- but don't play with themselves
        a.~a & h.~h in iden           -- no repeated games with exactly the same teams
        (a+h).~(a+h) & d.~d in iden   -- no team playing twice on same day
        s = a.~h & h.~a               -- game isomorphism (yields the "symmetric game")
                                      -- It is an isomorphism by construction: s = ~s
                                      -- and it is a function :-)

    }

    run{} for exactly 4 T, 12 G, 6 D -- why 12? 12 = 4*(4-1)
```