Formalizing Domain Models in FOL

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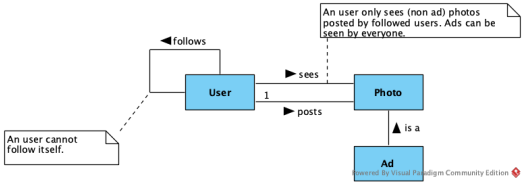
Domain models



Recall that a domain model is a diagram where

- entities are represented by boxes and
- relationships between entities by arcs annotated with
  - the name of the relationship/association,
  - the reading direction (indicated by an arrow) and
  - its multiplicity (annotated at the tip of the arcs).
- annotations with restrictions that are informally indicated in boxes may also appear in the diagram.

Domain models

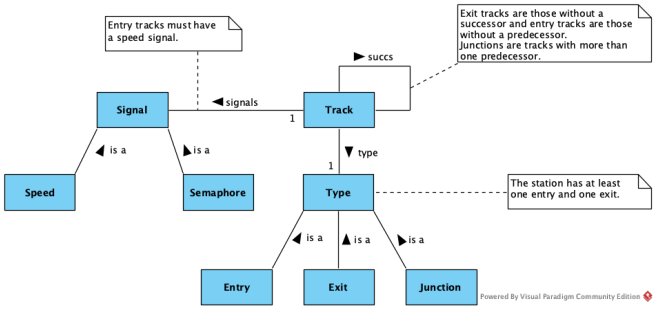
- There is no standard notation for domain models. We assume that:
  - all entities present in a diagram are disjoint, unless between two entities there is a relation/association *"is a"* which denotes a specialization/extension;
  - if there are multiple specializations for the same entity, those specializations are disjoint;
  - the relation/association *"is a"* may be a subset or a membership relation. If it is a membership relation, the entity that "belongs to" another will be modeled as a constant.

Formalizing a domain model

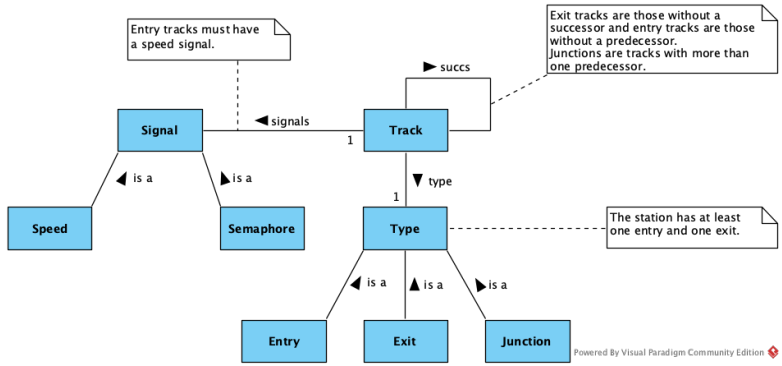
- First establish the logical language (i.e., the vocabulary).
  - a unary predicate for each entity (these predicates will act as the "type" of the entity, in a domain that is untyped);
  - a predicate for each association, except specializations (which will be codified by formulas that establish the kind of specialization);
  - a constant for each entity belonging to an "enumerated type".
- Next define the set of formulas that describe the system. They are of different nature:
  - codification of specialization relationships (which may be subset or membership relations, depending on the case);
  - partitioning the universe of discourse with the types of the entities;
  - disjunction of specialization entities;
  - typing associations;
  - multiplicity restrictions on associations;
  - restrictions annotated informally.

Example: Train station

In simplistic terms, the layout of a train station is composed of tracks that are connected to each other. A track can have more than one successor, namely if it ends in a railway switch. A switch can also be used to form a junction between two or more tracks. Trains arrive at the station through entry tracks and leave the station through exit tracks. The interlocking system is responsible for ensuring the safe flow of the trains through a station. A key part of the interlocking are signals. Possible signals are semaphores and speed limits.



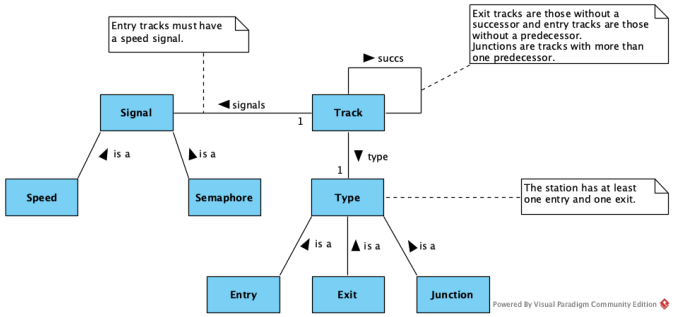
Example: Train station



Predicates: Signal(-) Speed(-) Semaphore(-) Track(-) Type(-) signals(-,-) type(-,-) succs(-,-)

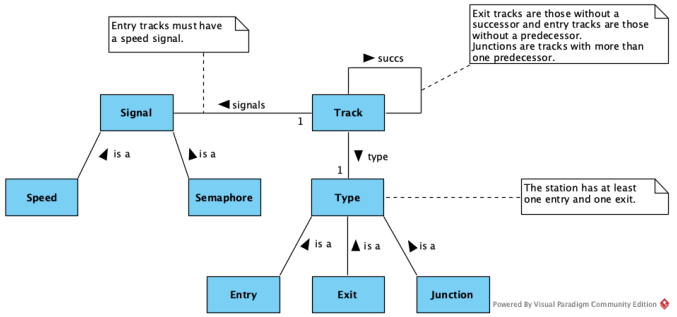
Constants: Entry Exit Junction

Example: Train station



- Codification of specialization relationships.
  - The Signal entity has two subsets.
    $\forall x.\, \mathsf{Speed}(x) \rightarrow \mathsf{Signal}(x)$
    $\forall x.\, \mathsf{Semaphore}(x) \rightarrow \mathsf{Signal}(x)$
  - The Type entity is an "enumerated type" with at least 3 elements.
    $\mathsf{Type}(\mathsf{Entry})$, $\mathsf{Type}(\mathsf{Exit})$, $\mathsf{Type}(\mathsf{Junction})$

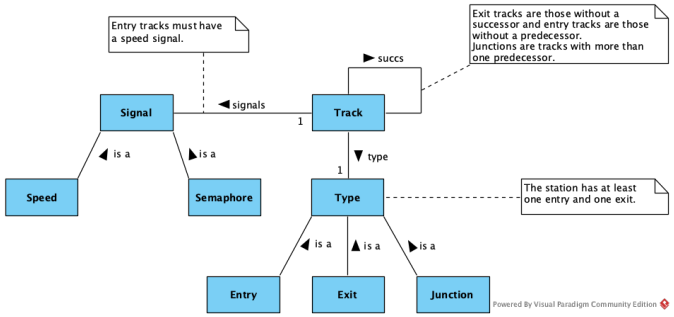
Example: Train station



- Partitioning the universe of discourse with the types of the entities.

$$\forall x.\, \mathsf{Signal}(x) \leftrightarrow \neg\mathsf{Track}(x) \wedge \neg\mathsf{Type}(x)$$
$$\forall x.\, \mathsf{Track}(x) \leftrightarrow \neg\mathsf{Signal}(x) \wedge \neg\mathsf{Type}(x)$$
$$\forall x.\, \mathsf{Type}(x) \leftrightarrow \neg\mathsf{Track}(x) \wedge \neg\mathsf{Signal}(x)$$

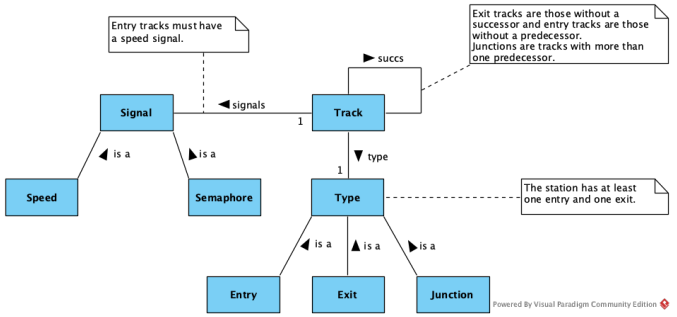
Example: Train station



- Disjunction of specialization entities.

$$\forall x.\, \mathsf{Speed}(x) \rightarrow \neg \mathsf{Semaphore}(x)$$
$$\mathsf{Entry} \neq \mathsf{Exit} \wedge \mathsf{Entry} \neq \mathsf{Junction} \wedge \mathsf{Exit} \neq \mathsf{Junction}$$

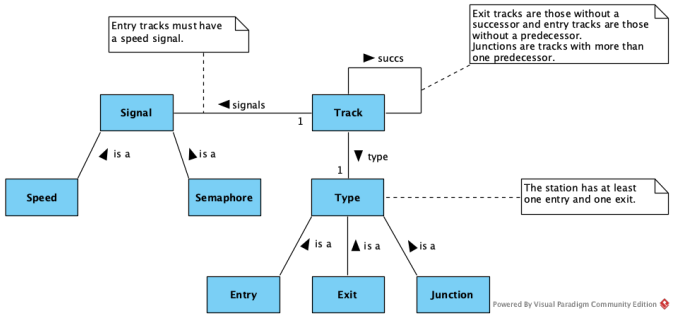
Example: Train station



- Typing associations.

$$\forall x, y.\, \mathsf{signals}(x, y) \rightarrow \mathsf{Track}(x) \wedge \mathsf{Signal}(y)$$
$$\forall x, y.\, \mathsf{succs}(x, y) \rightarrow \mathsf{Track}(x) \wedge \mathsf{Track}(y)$$
$$\forall x, y.\, \mathsf{type}(x, y) \rightarrow \mathsf{Track}(x) \wedge \mathsf{Type}(y)$$

Example: Train station

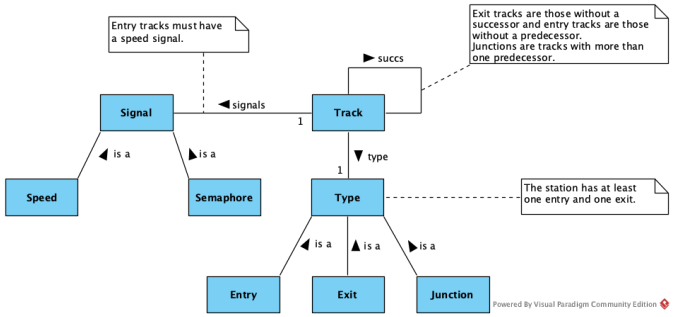


- Multiplicity restrictions on associations.

$$(\forall x.\, \mathsf{Signal}(x) \rightarrow \exists y.\, \mathsf{signals}(y, x)) \;\wedge$$
$$(\forall x, y, z.\, \mathsf{signals}(x, z) \wedge \mathsf{signals}(y, z) \rightarrow x = y)$$

$$(\forall x.\, \mathsf{Track}(x) \rightarrow \exists y.\, \mathsf{type}(x, y)) \;\wedge$$
$$(\forall x, y, z.\, \mathsf{type}(x, z) \wedge \mathsf{type}(x, y) \rightarrow z = y)$$

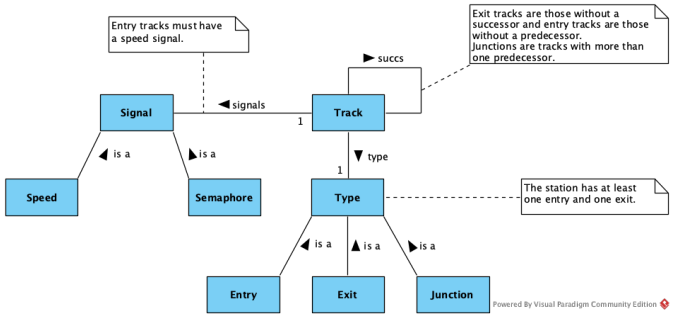
Example: Train station



- Restrictions annotated informally.
  - *Entry tracks must have a speed signal.*

$$\forall x.\, \mathsf{type}(x, \mathsf{Entry}) \rightarrow \exists y.\, \mathsf{signals}(x, y) \wedge \mathsf{Speed}(y)$$

Example: Train station



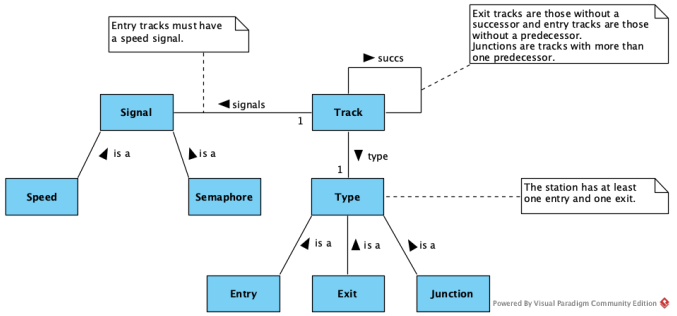
- *Exit tracks are those without a successor and entry tracks are those without a predecessor. Junctions are tracks with more than one predecessor.*

$$\forall x.\, \text{type}(x, \text{Exit}) \leftrightarrow \forall y.\, \neg\text{succs}(x, y)$$

$$\forall x.\, \text{type}(x, \text{Entry}) \leftrightarrow \neg\exists y.\, \text{succs}(y, x)$$

$$\forall x.\, \text{type}(x, \text{Junction}) \leftrightarrow \exists y, z.\, \text{succs}(y, x) \wedge \text{succs}(z, x) \wedge y \neq z$$

Example: Train station

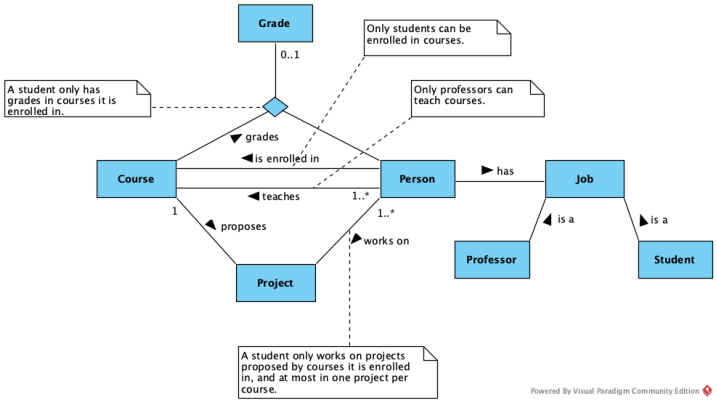


- *The station has at least one entry and one exit.*

$$\exists x.\, \text{type}(x, \text{Entry})$$

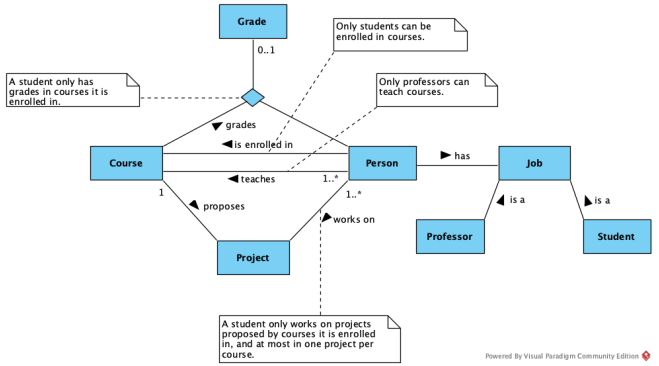
$$\exists x.\, \text{type}(x, \text{Exit})$$

Example: Courses



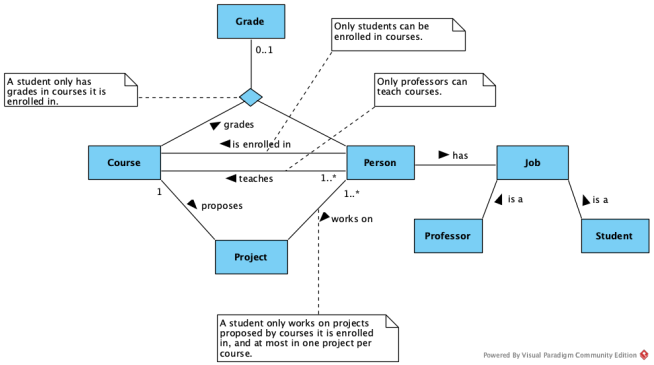
In this diagram, the diamond symbol is used to represent an n-ary association. In this case *grades* will be a ternary predicate that associates *courses* with *persons* and *grades*.

Example: Courses



Predicates: Course(-) Grade(-) Person(-) Project(-)
Job(-) Professor(-) Student(-)
enrolledin(-,-) teaches(-,-) has(-,-)
porposes(-,-) workson(-,-) grades(-,-,-)

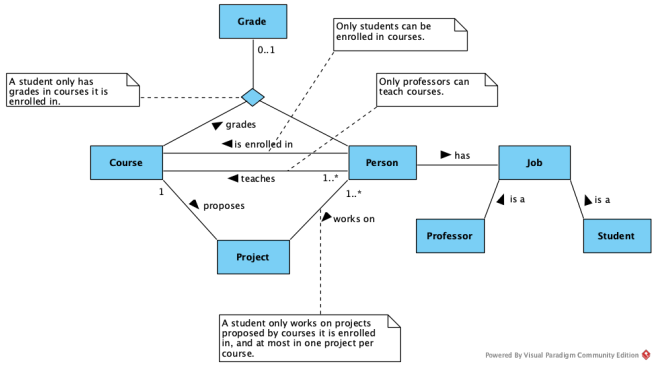
Example: Courses



- Codification of specialization relationships.
  - The Job entity has two subsets.

$$\forall x.\, \mathsf{Professor}(x) \rightarrow \mathsf{Job}(x)$$
$$\forall x.\, \mathsf{Student}(x) \rightarrow \mathsf{Job}(x)$$

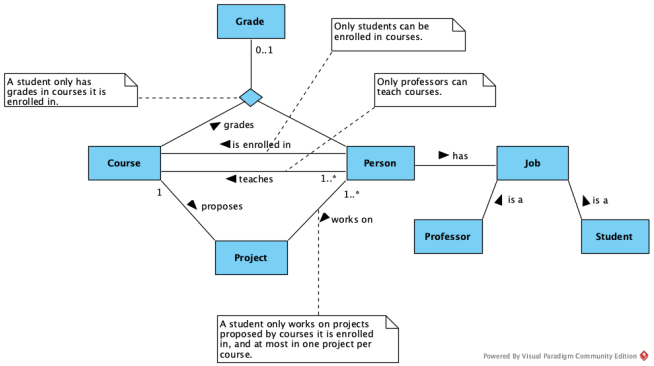
Example: Courses



- Partitioning the universe of discourse with the types of the entities.

$$\forall x.\, \mathsf{Course}(x) \leftrightarrow \neg\mathsf{Project}(x) \wedge \neg\mathsf{Grade}(x) \wedge \neg\mathsf{Person}(x) \wedge \neg\mathsf{Job}(x)$$
$$\forall x.\, \mathsf{Project}(x) \leftrightarrow \neg\mathsf{Course}(x) \wedge \neg\mathsf{Grade}(x) \wedge \neg\mathsf{Person}(x) \wedge \neg\mathsf{Job}(x)$$
$$\forall x.\, \mathsf{Grade}(x) \leftrightarrow \neg\mathsf{Project}(x) \wedge \neg\mathsf{Course}(x) \wedge \neg\mathsf{Person}(x) \wedge \neg\mathsf{Job}(x)$$
$$\forall x.\, \mathsf{Job}(x) \leftrightarrow \neg\mathsf{Project}(x) \wedge \neg\mathsf{Grade}(x) \wedge \neg\mathsf{Person}(x) \wedge \neg\mathsf{Course}(x)$$

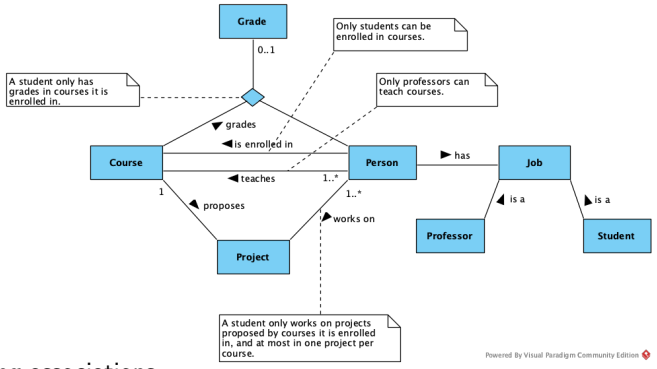
Example: Courses



- Disjunction of specialization entities.

$$\forall x.\, \mathsf{Professor}(x) \rightarrow \neg\mathsf{Student}(x)$$

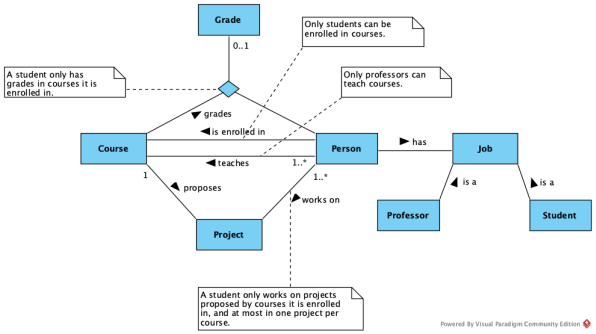
Example: Courses



- Typing associations.

$$\forall x, y, z.\, \mathsf{grades}(x, y, z) \rightarrow \mathsf{Person}(x) \wedge \mathsf{Course}(y) \wedge \mathsf{Grade}(z)$$
$$\forall x, y.\, \mathsf{enrolledin}(x, y) \rightarrow \mathsf{Person}(x) \wedge \mathsf{Course}(y)$$
$$\forall x, y.\, \mathsf{porposes}(x, y) \rightarrow \mathsf{Course}(x) \wedge \mathsf{Project}(y)$$
$$\forall x, y.\, \mathsf{workson}(x, y) \rightarrow \mathsf{Person}(x) \wedge \mathsf{Project}(y)$$
$$\forall x, y.\, \mathsf{teaches}(x, y) \rightarrow \mathsf{Person}(x) \wedge \mathsf{Course}(y)$$
$$\forall x, y.\, \mathsf{has}(x, y) \rightarrow \mathsf{Person}(x) \wedge \mathsf{Job}(y)$$

Example: Courses



- Multiplicity restrictions on associations.

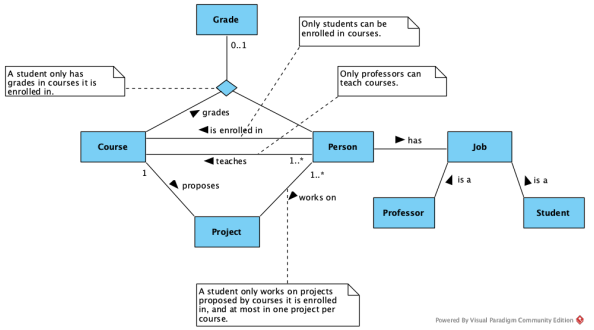
$$(\forall x.\, \mathsf{Project}(x) \rightarrow \exists y.\, \mathsf{porposes}(y, x)) \;\wedge$$
$$(\forall x, y, z.\, \mathsf{porposes}(x, z) \wedge \mathsf{porposes}(y, z) \rightarrow x = y)$$

$$\forall x.\, \mathsf{Course}(x) \rightarrow \exists y.\, \mathsf{teaches}(y, x)$$

$$\forall x.\, \mathsf{Project}(x) \rightarrow \exists y.\, \mathsf{workson}(y, x)$$

$$\forall x, y, a, b.\, \mathsf{grades}(x, y, a) \wedge \mathsf{grades}(x, y, b) \rightarrow a = b$$

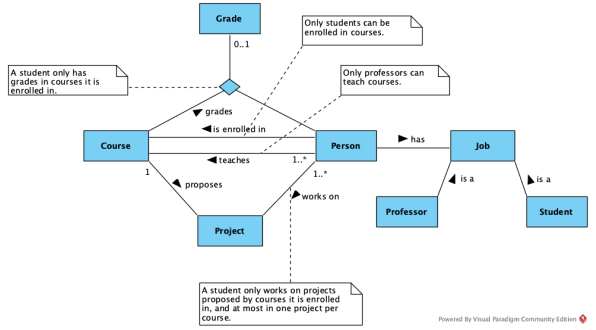
Example: Courses



- Restrictions annotated informally.
  - *A student only has grades in courses it is enrolled in.*

$$\forall x, y, z, j.\, \mathsf{has}(x, j) \wedge \mathsf{Student}(j) \wedge \mathsf{grades}(x, y, z) \rightarrow \mathsf{enrolledin}(x, y)$$

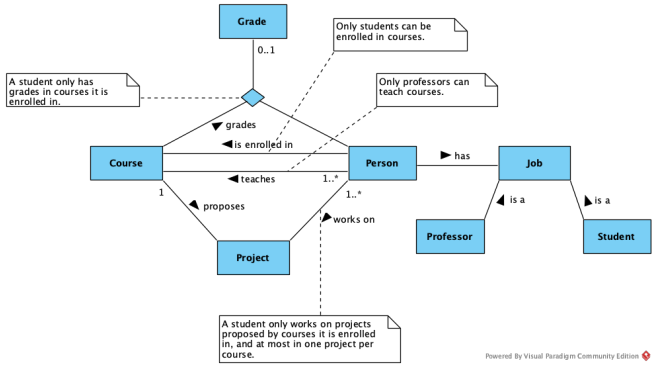
Example: Courses



- *Only students can be enrolled in courses.*

$$\forall x, y.\, \mathsf{enrolledin}(x, y) \rightarrow \exists j.\, \mathsf{has}(x, j) \wedge \mathsf{Student}(j)$$

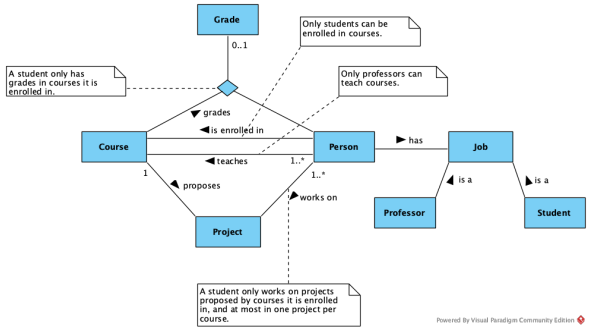
Example: Courses



- *Only professors can teach courses.*

$$\forall x, y.\, \mathsf{teaches}(x, y) \rightarrow \exists j.\, \mathsf{has}(x, j) \wedge \mathsf{Professor}(j)$$

Example: Courses



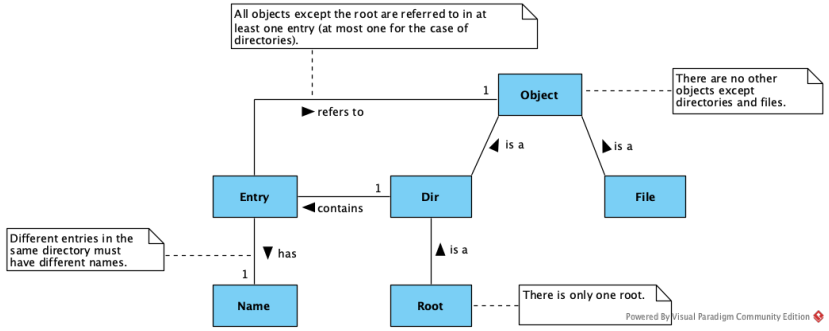
- *A student only works on projects proposed by courses it is enrolled in, and at most in one project per course.*

$$(\forall x, y.\, \text{workson}(x, y) \land (\exists j.\, \text{has}(x, j) \land \text{Student}(j)) \rightarrow \exists z.\, \text{porposes}(z, y) \land \text{enrolledin}(x, z))$$

$$\land$$

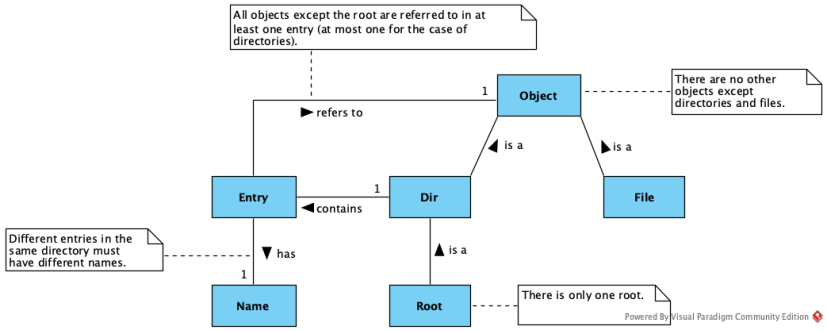
$$(\forall x, y, z, w.\, \text{workson}(x, y) \land \text{workson}(x, z) \land (\exists j.\, \text{has}(x, j) \land \text{Student}(j)) \land \text{porposes}(w, y) \land \text{porposes}(w, z) \rightarrow y = z)$$

Example: File system



Predicates: Entry(-) Object(-) Dir(-)
File(-) Name(-) Root(-)
refersto(-,-) has(-,-) contains(-,-)

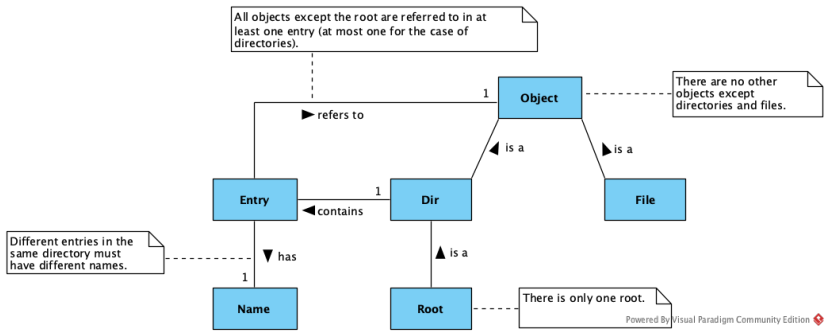
Example: File system



- Codification of specialization relationships.

$$\forall x.\, \text{Dir}(x) \rightarrow \text{Object}(x)$$
$$\forall x.\, \text{File}(x) \rightarrow \text{Object}(x)$$
$$\forall x.\, \text{Root}(x) \rightarrow \text{Dir}(x)$$

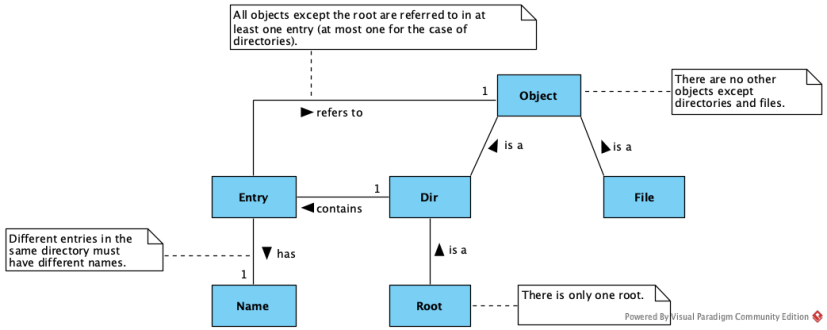
Example: File system



- Partitioning the universe of discourse with the types of the entities.

$$\forall x.\, \text{Entry}(x) \leftrightarrow \neg\text{Name}(x) \land \neg\text{Object}(x)$$
$$\forall x.\, \text{Name}(x) \leftrightarrow \neg\text{Entry}(x) \land \neg\text{Object}(x)$$
$$\forall x.\, \text{Object}(x) \leftrightarrow \neg\text{Entry}(x) \land \neg\text{Name}(x)$$

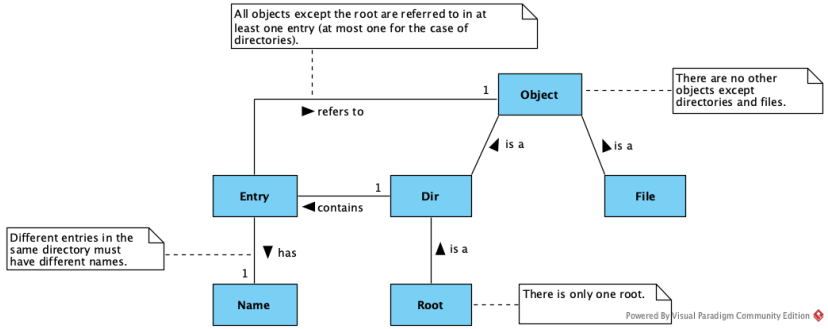
Example: File system



- Disjunction of specialization entities.

$$\forall x.\, \mathsf{Dir}(x) \rightarrow \neg \mathsf{File}(x)$$

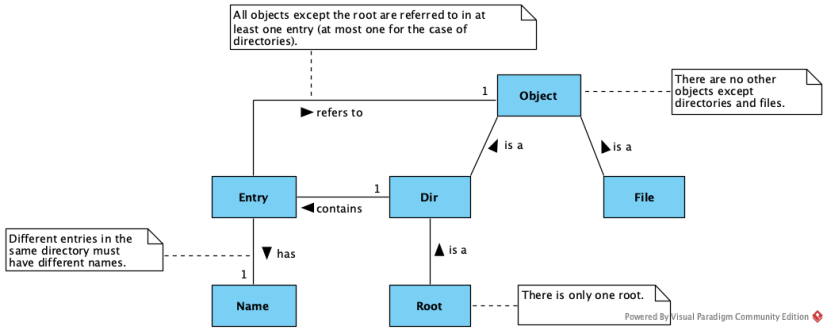
Example: File system



- Typing associations.

$$\forall x, y.\, \mathsf{has}(x, y) \rightarrow \mathsf{Entry}(x) \wedge \mathsf{Name}(y)$$
$$\forall x, y.\, \mathsf{refersto}(x, y) \rightarrow \mathsf{Entry}(x) \wedge \mathsf{Object}(y)$$
$$\forall x, y.\, \mathsf{contains}(x, y) \rightarrow \mathsf{Dir}(x) \wedge \mathsf{Entry}(y)$$

Example: File system



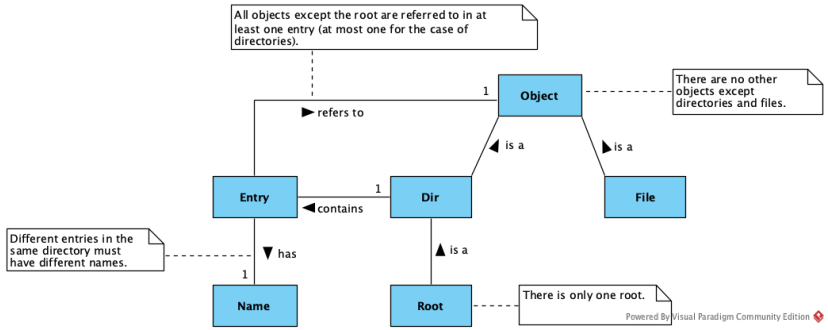
- Multiplicity restrictions on associations.

$$(\forall x.\, \mathsf{Entry}(x) \rightarrow \exists y.\, \mathsf{has}(x, y)) \wedge (\forall x, y, z.\, \mathsf{has}(x, y) \wedge \mathsf{has}(x, z) \rightarrow y = z)$$

$$(\forall x.\, \mathsf{Entry}(x) \rightarrow \exists y.\, \mathsf{refersto}(x, y)) \wedge (\forall x, y, z.\, \mathsf{refersto}(x, y) \wedge \mathsf{refersto}(x, z) \rightarrow y = z)$$

$$(\forall x.\, \mathsf{Entry}(x) \rightarrow \exists y.\, \mathsf{contains}(y, x)) \wedge (\forall x, y, z.\, \mathsf{contains}(x, y) \wedge \mathsf{contains}(x, z) \rightarrow y = z)$$

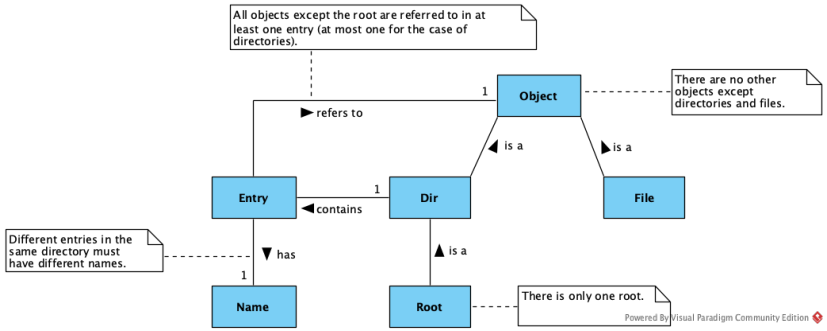
Example: File system



- Restrictions annotated informally.
  - *Different entries in the same directory must have different names.*

$$\forall x, y, z, n.\, \mathsf{contains}(x, y) \wedge \mathsf{contains}(x, z) \wedge y \neq z \wedge \mathsf{has}(y, n) \rightarrow \neg \mathsf{has}(z, n)$$

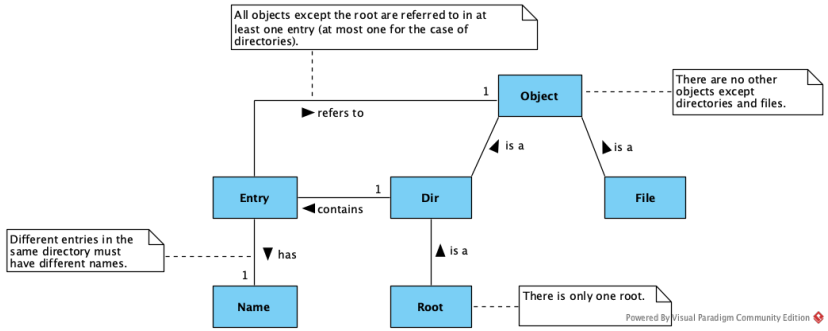
Example: File system



- *There is only one root.*

$$\forall x, y.\, \mathsf{Root}(x) \wedge \mathsf{Root}(y) \rightarrow x = y$$

Example: File system



- *There are no other objects except directories and files.*

$$\forall x.\, \mathsf{Object}(x) \rightarrow \mathsf{Dir}(x) \vee \mathsf{File}(x)$$