Leis do Cálculo Funcional (2025/26)

FUNÇÕES

Natural-id	$f \cdot id = id \cdot f = f$	(1)
Assoc-comp	$(f \cdot g) \cdot h \quad = \quad f \cdot (g \cdot h)$	(2)
Fusão-const	$\underline{k} \cdot f \quad = \quad \underline{k}$	(3)
Absorção-const	$f \cdot \underline{k} \quad = \quad \underline{f\ k}$	(4)
Eq-const	$a = b \quad \equiv \quad \underline{a} = \underline{b}$	(5)
Leibniz	$\left\{ \begin{array}{l} f \cdot h = g \cdot h \\ h \cdot f = h \cdot g \end{array} \right. \quad \Leftarrow \quad f = g$	(6)

PRODUTO

Universal-$\times$	$k = \langle f, g \rangle \quad \Leftrightarrow \quad \left\{ \begin{array}{l} \pi_1 \cdot k = f \\ \pi_2 \cdot k = g \end{array} \right.$	(7)
Cancelamento-$\times$	$\left\{ \begin{array}{l} \pi_1 \cdot \langle f, g \rangle = f \\ \pi_2 \cdot \langle f, g \rangle = g \end{array} \right.$	(8)
Reflexão-$\times$	$\langle \pi_1, \pi_2 \rangle \quad = \quad id_{A \times B}$	(9)
Fusão-$\times$	$\langle g, h \rangle \cdot f = \langle g \cdot f, h \cdot f \rangle$	(10)
Def-$\times$	$f \times g = \langle f \cdot \pi_1, g \cdot \pi_2 \rangle$	(11)
Absorção-$\times$	$(i \times j) \cdot \langle g, h \rangle = \langle i \cdot g, j \cdot h \rangle$	(12)
Natural-π_1	$\pi_1 \cdot (f \times g) \quad = \quad f \cdot \pi_1$	(13)
Natural-π_2	$\pi_2 \cdot (f \times g) \quad = \quad g \cdot \pi_2$	(14)
Functor-$\times$	$(g \cdot h) \times (i \cdot j) \quad = \quad (g \times i) \cdot (h \times j)$	(15)
Functor-id-$\times$	$id_A \times id_B \quad = \quad id_{A \times B}$	(16)
Eq-$\times$	$\langle f, g \rangle = \langle h, k \rangle \ \Leftrightarrow \ \left\{ \begin{array}{l} f = h \\ g = k \end{array} \right.$	(17)

COPRODUTO

Universal-$+$	$k = [f\ , g] \quad \Leftrightarrow \quad \left\{ \begin{array}{l} k \cdot i_1 = f \\ k \cdot i_2 = g \end{array} \right.$	(18)
Cancelamento-$+$	$\left\{ \begin{array}{l} [f\ , g] \cdot i_1 = f \\ [f\ , g] \cdot i_2 = g \end{array} \right.$	(19)
Reflexão-$+$	$[i_1\ , i_2] \quad = \quad id_{A+B}$	(20)
Fusão-$+$	$f \cdot [g\ , h] = [f \cdot g\ , f \cdot h]$	(21)
Def-$+$	$f + g = [i_1 \cdot f\ , i_2 \cdot g]$	(22)
Absorção-$+$	$[g\ , h] \cdot (i + j) = [g \cdot i\ , h \cdot j]$	(23)
Natural-i_1	$(i + j) \cdot i_1 \quad = \quad i_1 \cdot i$	(24)
Natural-i_2	$(i + j) \cdot i_2 \quad = \quad i_2 \cdot j$	(25)
Functor-$+$	$(g \cdot h) + (i \cdot j) = (g + i) \cdot (h + j)$	(26)
Functor-id-$+$	$id_A + id_B = id_{A+B}$	(27)
Eq-$+$	$[f\ , g] = [h\ , k] \ \Leftrightarrow \ \left\{ \begin{array}{l} f = h \\ g = k \end{array} \right.$	(28)

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Lei da troca $$[\langle f, g\rangle , \langle h, k\rangle] = \langle [f , h], [g , k]\rangle \quad (29)$$

CONDICIONAL

Natural-guarda	$p? \cdot f = (f + f) \cdot (p \cdot f)?$	(30)
Def condicional de McCarthy	$p \rightarrow f, g = [f , g] \cdot p?$	(31)
1.ª Lei de fusão do condicional	$f \cdot (p \rightarrow g, h) = p \rightarrow f \cdot g, f \cdot h$	(32)
2.ª Lei de fusão do condicional	$(p \rightarrow f, g) \cdot h = (p \cdot h) \rightarrow (f \cdot h), (g \cdot h)$	(33)

ISOMORFISMOS (α)

'Shunt-left'	$h \cdot \alpha = k \equiv h = k \cdot \alpha^\circ$	(34)
'Shunt-right'	$\alpha \cdot g = f \equiv g = \alpha^\circ \cdot f$	(35)

EXPONENCIAÇÃO

Universal-exp	$k = \overline{f} \Leftrightarrow f = \mathsf{ap} \cdot (k \times id)$	(36)
Cancelamento-exp	$f = \mathsf{ap} \cdot (\overline{f} \times id)$	(37)
Reflexão-exp	$\overline{\mathsf{ap}} = id_{B^A}$	(38)
Fusão-exp	$\overline{g \cdot (f \times id)} = \overline{g} \cdot f$	(39)
Def-exp	$f^A = \overline{f \cdot \mathsf{ap}} = (f\cdot)$	(40)
Absorção-exp	$f^A \cdot \overline{g} = \overline{f \cdot g}$	(41)
Natural-exp	$g \cdot \mathsf{ap} = \mathsf{ap} \cdot (g^A \times id)$	(42)
Functor-exp	$(g \cdot h)^A = g^A \cdot h^A$	(43)
Functor-id-exp	$id^A = id$	(44)

FUNCTORES

Functor-F	$\mathsf{F}(g \cdot h) = (\mathsf{F}\, g) \cdot (\mathsf{F}\, h)$	(45)
Functor-id-F	$\mathsf{F}\, id_A = id_{(\mathsf{F}\, A)}$	(46)

INDUÇÃO

Universal-cata	$k = (\!\lvert\, g \,\rvert\!) \Leftrightarrow k \cdot \mathsf{in} = g \cdot \mathsf{F}\, k$	(47)
Cancelamento-cata	$(\!\lvert\, g \,\rvert\!) \cdot \mathsf{in} = g \cdot \mathsf{F}\, (\!\lvert\, g \,\rvert\!)$	(48)
Reflexão-cata	$(\!\lvert\, \mathsf{in} \,\rvert\!) = id_{\mathsf{T}}$	(49)
Fusão-cata	$f \cdot (\!\lvert\, g \,\rvert\!) = (\!\lvert\, h \,\rvert\!) \Leftarrow f \cdot g = h \cdot \mathsf{F}\, f$	(50)
Base-cata	$\mathsf{F}\, f = \mathsf{B}\,(id, f)$	(51)
Def-map-cata	$\mathsf{T}\, f = (\!\lvert \mathsf{in} \cdot \mathsf{B}(f, id) \rvert\!)$	(52)
Absorção-cata	$(\!\lvert g \rvert\!) \cdot \mathsf{T}\, f = (\!\lvert g \cdot \mathsf{B}(f, id) \rvert\!)$	(53)

Recursividade mútua

Fokkinga $$\left\{\begin{array}{l} f \cdot in = h \cdot \mathsf{F}\,\langle f,g\rangle \\ g \cdot in = k \cdot \mathsf{F}\,\langle f,g\rangle \end{array}\right. \quad\equiv\quad \langle f,g\rangle = (\!|\langle h,k\rangle|\!) \tag{54}$$

"Banana-split" $$\langle (\!|i|\!), (\!|j|\!)\rangle \quad=\quad (\!|(i \times j)\cdot\langle \mathsf{F}\,\pi_1, \mathsf{F}\,\pi_2\rangle|\!) \tag{55}$$

Coindução

Universal-ana $$k = [\!(g)\!] \quad\Leftrightarrow\quad \mathsf{out}\cdot k = (\mathsf{F}\,k)\cdot g \tag{56}$$

Cancelamento-ana $$\mathsf{out}\cdot[\!(g)\!] = \mathsf{F}\,[\!(g)\!]\cdot g \tag{57}$$

Reflexão-ana $$[\!(\mathsf{out})\!] = id_{\mathsf{T}} \tag{58}$$

Fusão-ana $$[\!(g)\!]\cdot f = [\!(h)\!] \quad\Leftarrow\quad g\cdot f = (\mathsf{F}\,f)\cdot h \tag{59}$$

Base-ana $$\mathsf{F}\,f \quad=\quad \mathsf{B}\,(id, f) \tag{60}$$

Def-map-ana $$\mathsf{T}\,f \quad=\quad [\!(\mathsf{B}(f, id)\cdot\mathsf{out})\!] \tag{61}$$

Absorção-ana $$\mathsf{T}\,f\cdot[\!(g)\!] \quad=\quad [\!(\mathsf{B}(f, id)\cdot g)\!] \tag{62}$$

Mónadas

Multiplicação $$\mu\cdot\mu = \mu\cdot\mathsf{T}\,\mu \tag{63}$$

Unidade $$\mu\cdot u = \mu\cdot\mathsf{T}\,u = id \tag{64}$$

Natural-u $$u\cdot f \quad=\quad \mathsf{T}\,f\cdot u \tag{65}$$

Natural-μ $$\mu\cdot\mathsf{T}\,(\mathsf{T}\,f) \quad=\quad \mathsf{T}\,f\cdot\mu \tag{66}$$

Composição monádica $$f\bullet g \quad=\quad \mu\cdot\mathsf{T}\,f\cdot g \tag{67}$$

Associatividade-$\bullet$ $$f\bullet(g\bullet h) = (f\bullet g)\bullet h \tag{68}$$

Identidade-$\bullet$ $$u\bullet f = f = f\bullet u \tag{69}$$

Associatividade-$\bullet/\cdot$ $$(f\bullet g)\cdot h \quad=\quad f\bullet(g\cdot h) \tag{70}$$

Associatividade-$\cdot/\bullet$ $$(f\cdot g)\bullet h \quad=\quad f\bullet(\mathsf{T}\,g\cdot h) \tag{71}$$

μ versus $\bullet$ $$id\bullet id \quad=\quad \mu \tag{72}$$

Definições *ao ponto* ('pointwise')

Igualdade extensional $$f = g \quad\Leftrightarrow\quad \langle\forall\, x \,::\, f\ x = g\ x\rangle \tag{73}$$

Def-comp $$(f\cdot g)\ x \quad=\quad f\ (g\ x) \tag{74}$$

Def-id $$id\,x \quad=\quad x \tag{75}$$

Def-const $$\underline{k}\ x \quad=\quad k \tag{76}$$

Notação-λ $$f\ a = b \quad\equiv\quad f = \lambda a \rightarrow b \tag{77}$$

Def-split $$\langle f, g\rangle\,x \quad=\quad (f\,x, g\,x) \tag{78}$$

Def-$\times$ $$(f\times g)\,(a,b) \quad=\quad (f\,a, g\,b) \tag{79}$$

Def-proj $$\left\{\begin{array}{l} \pi_1\ (x,y) = x \\ \pi_2\ (x,y) = y \end{array}\right. \tag{80}$$

Elim-let $$\textbf{let}\ x = a\ \textbf{in}\ b \quad=\quad b\,[x/a] \tag{81}$$

Elim-pair $$t \quad=\quad t[(x,y)/z, x/\pi_1 z, y/\pi_2 z] \tag{82}$$

Def-cond $$(p\rightarrow f, g)\,x \quad=\quad \textbf{if}\ p\ x\ \textbf{then}\ f\ x\ \textbf{else}\ g\ x \tag{83}$$

Def-guard $$p?\ a = \textbf{if}\ p\ a\ \textbf{then}\ i_1\ a\ \textbf{else}\ i_2\ a \tag{84}$$

Def-ap $$ap(f,x) = f\ x \tag{85}$$

Curry	$\overline{f}\ a\ b = f\ (a, b)$	(86)
Uncurry	$\widehat{f}\ (a, b) = f\ a\ b$	(87)
Composição monádica	$(f \bullet g)\ a = \mathbf{do}\ \{\, b \leftarrow g\ a; f\ b \,\}$	(88)
'Binding-μ'	$x \ggg= f \quad = \quad (\mu \cdot \mathsf{T}\, f) x$	(89)
Notação-do	$\mathbf{do}\ \{\, x \leftarrow a; b \,\} \quad = \quad a \ggg= (\lambda x \rightarrow b)$	(90)
'μ-binding'	$\mu\, x \quad = \quad x \ggg= id$	(91)
Sequenciação	$x \gg y \quad = \quad x \ggg= \underline{y}$	(92)