Leis do Cálculo Funcional (2024/25)

Funções

Natural-id	$f \cdot id = id \cdot f = f$	(1)
Assoc-comp	$(f \cdot g) \cdot h \quad = \quad f \cdot (g \cdot h)$	(2)
Fusão-const	$\underline{k} \cdot f \quad = \quad \underline{k}$	(3)
Absorção-const	$f \cdot \underline{k} \quad = \quad \underline{f\ k}$	(4)
Leibniz	$\left\{ \begin{array}{l} f \cdot h = g \cdot h \\ h \cdot f = h \cdot g \end{array} \right. \quad \Leftarrow \quad f = g$	(5)

Produto

Universal-$\times$	$k = \langle f, g \rangle \quad \Leftrightarrow \quad \left\{ \begin{array}{l} \pi_1 \cdot k = f \\ \pi_2 \cdot k = g \end{array} \right.$	(6)
Cancelamento-$\times$	$\left\{ \begin{array}{l} \pi_1 \cdot \langle f, g \rangle = f \\ \pi_2 \cdot \langle f, g \rangle = g \end{array} \right.$	(7)
Reflexão-$\times$	$\langle \pi_1, \pi_2 \rangle \quad = \quad id_{A \times B}$	(8)
Fusão-$\times$	$\langle g, h \rangle \cdot f = \langle g \cdot f, h \cdot f \rangle$	(9)
Def-$\times$	$f \times g = \langle f \cdot \pi_1, g \cdot \pi_2 \rangle$	(10)
Absorção-$\times$	$(i \times j) \cdot \langle g, h \rangle = \langle i \cdot g, j \cdot h \rangle$	(11)
Natural-π_1	$\pi_1 \cdot (f \times g) \quad = \quad f \cdot \pi_1$	(12)
Natural-π_2	$\pi_2 \cdot (f \times g) \quad = \quad g \cdot \pi_2$	(13)
Functor-$\times$	$(g \cdot h) \times (i \cdot j) \quad = \quad (g \times i) \cdot (h \times j)$	(14)
Functor-id-$\times$	$id_A \times id_B \quad = \quad id_{A \times B}$	(15)
Eq-$\times$	$\langle f, g \rangle = \langle h, k \rangle \ \Leftrightarrow \left\{ \begin{array}{l} f = h \\ g = k \end{array} \right.$	(16)

Coproduto

Universal-$+$	$k = [f\ , g] \quad \Leftrightarrow \quad \left\{ \begin{array}{l} k \cdot i_1 = f \\ k \cdot i_2 = g \end{array} \right.$	(17)
Cancelamento-$+$	$\left\{ \begin{array}{l} [f\ , g] \cdot i_1 = f \\ [f\ , g] \cdot i_2 = g \end{array} \right.$	(18)
Reflexão-$+$	$[i_1\ , i_2] \quad = \quad id_{A+B}$	(19)
Fusão-$+$	$f \cdot [g\ , h] = [f \cdot g\ , f \cdot h]$	(20)
Def-$+$	$f + g = [i_1 \cdot f\ , i_2 \cdot g]$	(21)
Absorção-$+$	$[g\ , h] \cdot (i + j) = [g \cdot i\ , h \cdot j]$	(22)
Natural-i_1	$(i + j) \cdot i_1 \quad = \quad i_1 \cdot i$	(23)
Natural-i_2	$(i + j) \cdot i_2 \quad = \quad i_2 \cdot j$	(24)
Functor-$+$	$(g \cdot h) + (i \cdot j) = (g + i) \cdot (h + j)$	(25)
Functor-id-$+$	$id_A + id_B = id_{A+B}$	(26)
Eq-$+$	$[f\ , g] = [h\ , k] \ \Leftrightarrow \left\{ \begin{array}{l} f = h \\ g = k \end{array} \right.$	(27)

MISC. PRODUTO / COPRODUTO

Lei da troca	$[\langle f, g\rangle , \langle h, k\rangle] \quad = \quad \langle [f\ , h], [g\ , k]\rangle$	(28)

CONDICIONAL

Natural-guarda	$p? \cdot f \quad = \quad (f + f) \cdot (p \cdot f)?$	(29)
Def condicional de McCarthy	$p \rightarrow f, g \quad = \quad [f\ , g] \cdot p?$	(30)
1.ª Lei de fusão do condicional	$f \cdot (p \rightarrow g, h) \quad = \quad p \rightarrow f \cdot g, f \cdot h$	(31)
2.ª Lei de fusão do condicional	$(p \rightarrow f, g) \cdot h \quad = \quad (p \cdot h) \rightarrow (f \cdot h), (g \cdot h)$	(32)

ISOMORFISMOS (α)

'Shunt-left'	$h \cdot \alpha = k \quad \equiv \quad h = k \cdot \alpha^\circ$	(33)
'Shunt-right'	$\alpha \cdot g = f \quad \equiv \quad g = \alpha^\circ \cdot f$	(34)

EXPONENCIAÇÃO

Universal-exp	$k = \overline{f} \quad \Leftrightarrow \quad f = \mathsf{ap} \cdot (k \times id)$	(35)
Cancelamento-exp	$f = \mathsf{ap} \cdot (\overline{f} \times id)$	(36)
Reflexão-exp	$\overline{\mathsf{ap}} = id_{B^A}$	(37)
Fusão-exp	$\overline{g \cdot (f \times id)} = \overline{g} \cdot f$	(38)
Def-exp	$f^A = \overline{f \cdot \mathsf{ap}} = (f \cdot)$	(39)
Absorção-exp	$f^A \cdot \overline{g} = \overline{f \cdot g}$	(40)
Natural-exp	$g \cdot \mathsf{ap} = \mathsf{ap} \cdot (g^A \times id)$	(41)
Functor-exp	$(g \cdot h)^A = g^A \cdot h^A$	(42)
Functor-id-exp	$id^A = id$	(43)

FUNCTORES

Functor-F	$\mathsf{F}(g \cdot h) \quad = \quad (\mathsf{F}\, g) \cdot (\mathsf{F}\, h)$	(44)
Functor-id-F	$\mathsf{F}\, id_A \quad = \quad id_{(\mathsf{F}\, A)}$	(45)

INDUÇÃO

Universal-cata	$k = (\!|\, g \,|\!) \quad \Leftrightarrow \quad k \cdot \mathsf{in} = g \cdot \mathsf{F}\, k$	(46)
Cancelamento-cata	$(\!|\, g \,|\!) \cdot \mathsf{in} = g \cdot \mathsf{F}\, (\!|\, g \,|\!)$	(47)
Reflexão-cata	$(\!|\, \mathsf{in} \,|\!) = id_{\mathsf{T}}$	(48)
Fusão-cata	$f \cdot (\!|\, g \,|\!) = (\!|\, h \,|\!) \quad \Leftarrow \quad f \cdot g = h \cdot \mathsf{F}\, f$	(49)
Base-cata	$\mathsf{F}\, f \quad = \quad \mathsf{B}\, (id, f)$	(50)
Def-map-cata	$\mathsf{T}\, f \quad = \quad (\!|\mathsf{in} \cdot \mathsf{B}(f, id)|\!)$	(51)
Absorção-cata	$(\!|g|\!) \cdot \mathsf{T}\, f \quad = \quad (\!|g \cdot \mathsf{B}(f, id)|\!)$	(52)

Recursividade mútua

Fokkinga $$\left\{ \begin{array}{l} f \cdot in = h \cdot \mathsf{F}\,\langle f, g \rangle \\ g \cdot in = k \cdot \mathsf{F}\,\langle f, g \rangle \end{array} \right. \quad \equiv \quad \langle f, g \rangle = (\!|\langle h, k \rangle|\!) \tag{53}$$

"Banana-split" $$\langle (\!|i|\!), (\!|j|\!) \rangle \quad = \quad (\!|(i \times j) \cdot \langle \mathsf{F}\,\pi_1, \mathsf{F}\,\pi_2 \rangle|\!) \tag{54}$$

Coindução

Universal-ana $$k = [\!(g)\!] \quad \Leftrightarrow \quad \mathsf{out} \cdot k = (\mathsf{F}\,k) \cdot g \tag{55}$$

Cancelamento-ana $$\mathsf{out} \cdot [\!(g)\!] = \mathsf{F}\,[\!(g)\!] \cdot g \tag{56}$$

Reflexão-ana $$[\!(\mathsf{out})\!] = id_{\mathsf{T}} \tag{57}$$

Fusão-ana $$[\!(g)\!] \cdot f = [\!(h)\!] \quad \Leftarrow \quad g \cdot f = (\mathsf{F}\,f) \cdot h \tag{58}$$

Base-ana $$\mathsf{F}\,f \quad = \quad \mathsf{B}\,(id, f) \tag{59}$$

Def-map-ana $$\mathsf{T}\,f \quad = \quad [\!(\mathsf{B}(f, id) \cdot \mathsf{out})\!] \tag{60}$$

Absorção-ana $$\mathsf{T}\,f \cdot [\!(g)\!] \quad = \quad [\!(\mathsf{B}(f, id) \cdot g)\!] \tag{61}$$

Mónadas

Multiplicação $$\mu \cdot \mu = \mu \cdot \mathsf{T}\,\mu \tag{62}$$

Unidade $$\mu \cdot u = \mu \cdot \mathsf{T}\,u = id \tag{63}$$

Natural-u $$u \cdot f \quad = \quad \mathsf{T}\,f \cdot u \tag{64}$$

Natural-μ $$\mu \cdot \mathsf{T}\,(\mathsf{T}\,f) \quad = \quad \mathsf{T}\,f \cdot \mu \tag{65}$$

Composição monádica $$f \bullet g \quad = \quad \mu \cdot \mathsf{T}\,f \cdot g \tag{66}$$

Associatividade-$\bullet$ $$f \bullet (g \bullet h) = (f \bullet g) \bullet h \tag{67}$$

Identidade-$\bullet$ $$u \bullet f = f = f \bullet u \tag{68}$$

Associatividade-$\bullet/\cdot$ $$(f \bullet g) \cdot h \quad = \quad f \bullet (g \cdot h) \tag{69}$$

Associatividade-$\cdot/\bullet$ $$(f \cdot g) \bullet h \quad = \quad f \bullet (\mathsf{T}\,g \cdot h) \tag{70}$$

μ versus $\bullet$ $$id \bullet id \quad = \quad \mu \tag{71}$$

Definições *ao ponto* ('pointwise')

Igualdade extensional $$f = g \quad \Leftrightarrow \quad \langle \forall\, x \; :: \; f\ x = g\ x \rangle \tag{72}$$

Def-comp $$(f \cdot g)\ x \quad = \quad f\ (g\ x) \tag{73}$$

Def-id $$id\,x \quad = \quad x \tag{74}$$

Def-const $$\underline{k}\ x \quad = \quad k \tag{75}$$

Notação-λ $$f\ a = b \quad \equiv \quad f = \lambda a \to b \tag{76}$$

Def-split $$\langle f, g \rangle\, x \quad = \quad (f\,x, g\,x) \tag{77}$$

Def-$\times$ $$(f \times g)\,(a, b) \quad = \quad (f\,a, g\,b) \tag{78}$$

Def-proj $$\left\{ \begin{array}{l} \pi_1\ (x, y) = x \\ \pi_2\ (x, y) = y \end{array} \right. \tag{79}$$

Elim-let $$\mathbf{let}\ x = a\ \mathbf{in}\ b \quad = \quad b\,[x/a] \tag{80}$$

Elim-pair $$t \quad = \quad t[(x, y)/z, x/\pi_1 z, y/\pi_2 z] \tag{81}$$

Def-cond $$(p \to f, g)\,x \quad = \quad \mathbf{if}\ p\ x\ \mathbf{then}\ f\ x\ \mathbf{else}\ g\ x \tag{82}$$

Def-guard $$p?\ a = \mathbf{if}\ p\ a\ \mathbf{then}\ i_1\ a\ \mathbf{else}\ i_2\ a \tag{83}$$

Def-ap $$ap(f, x) = f\ x \tag{84}$$

Curry	$\overline{f}\ a\ b = f\ (a, b)$	(85)
Uncurry	$\widehat{f}\ (a, b) = f\ a\ b$	(86)
Composição monádica	$(f \bullet g)\ a = \mathbf{do}\ \{ b \leftarrow g\ a; f\ b \}$	(87)
'Binding-μ'	$x \gg\!= f \quad = \quad (\mu \cdot \mathsf{T}\, f)x$	(88)
Notação-do	$\mathbf{do}\ \{ x \leftarrow a; b \} \quad = \quad a \gg\!= (\lambda x \rightarrow b)$	(89)
'μ-binding'	$\mu\, x \quad = \quad x \gg\!= id$	(90)
Sequenciação	$x \gg y \quad = \quad x \gg\!= \underline{y}$	(91)