Leis do Cálculo Funcional (2023/24)

Funções

Natural-id	$f \cdot id = id \cdot f = f$	(1)
Assoc-comp	$(f \cdot g) \cdot h \quad = \quad f \cdot (g \cdot h)$	(2)
Natural-const	$\underline{k} \cdot f \quad = \quad \underline{k}$	(3)
Fusão-const	$f \cdot \underline{k} \quad = \quad \underline{f\ k}$	(4)
Leibniz	$\left\{ \begin{array}{l} f \cdot h = g \cdot h \\ h \cdot f = h \cdot g \end{array} \right. \quad \Leftarrow \quad f = g$	(5)

Produto

Universal-$\times$	$k = \langle f, g \rangle \quad \Leftrightarrow \quad \left\{ \begin{array}{l} \pi_1 \cdot k = f \\ \pi_2 \cdot k = g \end{array} \right.$	(6)
Cancelamento-$\times$	$\left\{ \begin{array}{l} \pi_1 \cdot \langle f, g \rangle = f \\ \pi_2 \cdot \langle f, g \rangle = g \end{array} \right.$	(7)
Reflexão-$\times$	$\langle \pi_1, \pi_2 \rangle \quad = \quad id_{A \times B}$	(8)
Fusão-$\times$	$\langle g, h \rangle \cdot f = \langle g \cdot f, h \cdot f \rangle$	(9)
Def-$\times$	$f \times g = \langle f \cdot \pi_1, g \cdot \pi_2 \rangle$	(10)
Absorção-$\times$	$(i \times j) \cdot \langle g, h \rangle = \langle i \cdot g, j \cdot h \rangle$	(11)
Natural-π_1	$\pi_1 \cdot (f \times g) \quad = \quad f \cdot \pi_1$	(12)
Natural-π_2	$\pi_2 \cdot (f \times g) \quad = \quad g \cdot \pi_2$	(13)
Functor-$\times$	$(g \cdot h) \times (i \cdot j) \quad = \quad (g \times i) \cdot (h \times j)$	(14)
Functor-id-$\times$	$id_A \times id_B \quad = \quad id_{A \times B}$	(15)
Eq-$\times$	$\langle f, g \rangle = \langle h, k \rangle \ \Leftrightarrow \left\{ \begin{array}{l} f = h \\ g = k \end{array} \right.$	(16)

Coproduto

Universal-$+$	$k = [f\ , g] \quad \Leftrightarrow \quad \left\{ \begin{array}{l} k \cdot i_1 = f \\ k \cdot i_2 = g \end{array} \right.$	(17)
Cancelamento-$+$	$\left\{ \begin{array}{l} [f\ , g] \cdot i_1 = f \\ [f\ , g] \cdot i_2 = g \end{array} \right.$	(18)
Reflexão-$+$	$[i_1\ , i_2] \quad = \quad id_{A+B}$	(19)
Fusão-$+$	$f \cdot [g\ , h] = [f \cdot g\ , f \cdot h]$	(20)
Def-$+$	$f + g = [i_1 \cdot f\ , i_2 \cdot g]$	(21)
Absorção-$+$	$[g\ , h] \cdot (i + j) = [g \cdot i\ , h \cdot j]$	(22)
Natural-i_1	$(i + j) \cdot i_1 \quad = \quad i_1 \cdot i$	(23)
Natural-i_2	$(i + j) \cdot i_2 \quad = \quad i_2 \cdot j$	(24)
Functor-$+$	$(g \cdot h) + (i \cdot j) = (g + i) \cdot (h + j)$	(25)
Functor-id-$+$	$id_A + id_B = id_{A+B}$	(26)
Eq-$+$	$[f\ , g] = [h\ , k] \ \Leftrightarrow \left\{ \begin{array}{l} f = h \\ g = k \end{array} \right.$	(27)

MISC. PRODUTO / COPRODUTO

Lei da troca	$[\langle f, g\rangle \,, \langle h, k\rangle] \quad = \quad \langle [f \,, h], [g \,, k]\rangle$	(28)

CONDICIONAL

Natural-guarda	$p? \cdot f \quad = \quad (f + f) \cdot (p \cdot f)?$	(29)
Def condicional de McCarthy	$p \rightarrow f, g \quad = \quad [f \,, g] \cdot p?$	(30)
1.ª Lei de fusão do condicional	$f \cdot (p \rightarrow g, h) \quad = \quad p \rightarrow f \cdot g, f \cdot h$	(31)
2.ª Lei de fusão do condicional	$(p \rightarrow f, g) \cdot h \quad = \quad (p \cdot h) \rightarrow (f \cdot h), (g \cdot h)$	(32)

ISOMORFISMOS (α)

'Shunt-left'	$h \cdot \alpha = k \quad \equiv \quad h = k \cdot \alpha^\circ$	(33)
'Shunt-right'	$\alpha \cdot g = f \quad \equiv \quad g = \alpha^\circ \cdot f$	(34)

EXPONENCIAÇÃO

Universal-exp	$k = \overline{f} \quad \Leftrightarrow \quad f = \mathsf{ap} \cdot (k \times id)$	(35)
Cancelamento-exp	$f = \mathsf{ap} \cdot (\overline{f} \times id)$	(36)
Reflexão-exp	$\overline{\mathsf{ap}} = id_{B^A}$	(37)
Fusão-exp	$\overline{g \cdot (f \times id)} = \overline{g} \cdot f$	(38)
Def-exp	$f^A = \overline{f \cdot \mathsf{ap}}$	(39)
Absorção-exp	$f^A \cdot \overline{g} = \overline{f \cdot g}$	(40)
Natural-exp	$g \cdot \mathsf{ap} = \mathsf{ap} \cdot (g^A \times id)$	(41)
Functor-exp	$(g \cdot h)^A = g^A \cdot h^A$	(42)
Functor-id-exp	$id^A = id$	(43)

FUNCTORES

Functor-F	$\mathsf{F}(g \cdot h) \quad = \quad (\mathsf{F}\, g) \cdot (\mathsf{F}\, h)$	(44)
Functor-id-F	$\mathsf{F}\, id_A \quad = \quad id_{(\mathsf{F}\, A)}$	(45)

INDUÇÃO

Universal-cata	$k = (\!\lvert\, g \,\rvert\!) \quad \Leftrightarrow \quad k \cdot \mathsf{in} = g \cdot \mathsf{F}\, k$	(46)
Cancelamento-cata	$(\!\lvert\, g \,\rvert\!) \cdot \mathsf{in} = g \cdot \mathsf{F}\, (\!\lvert\, g \,\rvert\!)$	(47)
Reflexão-cata	$(\!\lvert\, \mathsf{in} \,\rvert\!) = id_{\mathsf{T}}$	(48)
Fusão-cata	$f \cdot (\!\lvert\, g \,\rvert\!) = (\!\lvert\, h \,\rvert\!) \quad \Leftarrow \quad f \cdot g = h \cdot \mathsf{F}\, f$	(49)
Base-cata	$\mathsf{F}\, f \quad = \quad \mathsf{B}\, (id, f)$	(50)
Def-map-cata	$\mathsf{T}\, f \quad = \quad (\!\lvert\, \mathsf{in} \cdot \mathsf{B}(f, id) \,\rvert\!)$	(51)
Absorção-cata	$(\!\lvert\, g \,\rvert\!) \cdot \mathsf{T}\, f \quad = \quad (\!\lvert\, g \cdot \mathsf{B}(f, id) \,\rvert\!)$	(52)

Recursividade mútua

Fokkinga	$\left\{ \begin{array}{l} f \cdot in = h \cdot \mathsf{F}\,\langle f, g\rangle \\ g \cdot in = k \cdot \mathsf{F}\,\langle f, g\rangle \end{array} \right. \quad \equiv \quad \langle f, g\rangle = (\!\vert \langle h, k\rangle \vert\!)$	(53)
"Banana-split"	$\langle (\!\vert i \vert\!), (\!\vert j \vert\!)\rangle \quad = \quad (\!\vert (i \times j) \cdot \langle \mathsf{F}\,\pi_1, \mathsf{F}\,\pi_2\rangle \vert\!)$	(54)

Coindução

Universal-ana	$k = [\!(g)\!] \quad \Leftrightarrow \quad \mathsf{out} \cdot k = (\mathsf{F}\,k) \cdot g$	(55)
Cancelamento-ana	$\mathsf{out} \cdot [\!(g)\!] = \mathsf{F}\,[\!(g)\!] \cdot g$	(56)
Reflexão-ana	$[\!(\mathsf{out})\!] = id_{\mathsf{T}}$	(57)
Fusão-ana	$[\!(g)\!] \cdot f = [\!(h)\!] \quad \Leftarrow \quad g \cdot f = (\mathsf{F}\,f) \cdot h$	(58)
Base-ana	$\mathsf{F}\,f \quad = \quad \mathsf{B}\,(id, f)$	(59)
Def-map-ana	$\mathsf{T}\,f \quad = \quad [\!(\mathsf{B}(f, id) \cdot \mathsf{out})\!]$	(60)
Absorção-ana	$\mathsf{T}\,f \cdot [\!(g)\!] \quad = \quad [\!(\mathsf{B}(f, id) \cdot g)\!]$	(61)

Mónadas

Multiplicação	$\mu \cdot \mu = \mu \cdot \mathsf{T}\,\mu$	(62)
Unidade	$\mu \cdot u = \mu \cdot \mathsf{T}\,u = id$	(63)
Natural-u	$u \cdot f \quad = \quad \mathsf{T}\,f \cdot u$	(64)
Natural-μ	$\mu \cdot \mathsf{T}\,(\mathsf{T}\,f) \quad = \quad \mathsf{T}\,f \cdot \mu$	(65)
Composição monádica	$f \bullet g \quad = \quad \mu \cdot \mathsf{T}\,f \cdot g$	(66)
Associatividade-$\bullet$	$f \bullet (g \bullet h) = (f \bullet g) \bullet h$	(67)
Identidade-$\bullet$	$u \bullet f = f = f \bullet u$	(68)
Associatividade-$\bullet/\cdot$	$(f \bullet g) \cdot h \quad = \quad f \bullet (g \cdot h)$	(69)
Associatividade-$\cdot/\bullet$	$(f \cdot g) \bullet h \quad = \quad f \bullet (\mathsf{T}\,g \cdot h)$	(70)
μ **versus** $\bullet$	$id \bullet id \quad = \quad \mu$	(71)

Definições *ao ponto* ('pointwise')

Igualdade extensional	$f = g \quad \Leftrightarrow \quad \langle \forall\, x \,::\, f\ x = g\ x\rangle$	(72)
Def-comp	$(f \cdot g)\ x \quad = \quad f\ (g\ x)$	(73)
Def-id	$id\,x \quad = \quad x$	(74)
Def-const	$\underline{k}\ x \quad = \quad k$	(75)
Notação-λ	$f\ a = b \quad \equiv \quad f = \lambda a \rightarrow b$	(76)
Def-split	$\langle f, g\rangle\,x \quad = \quad (f\,x, g\,x)$	(77)
Def-$\times$	$(f \times g)\,(a, b) \quad = \quad (f\,a, g\,b)$	(78)
Def-proj	$\left\{ \begin{array}{l} \pi_1\ (x, y) = x \\ \pi_2\ (x, y) = y \end{array} \right.$	(79)
Elim-let	$\textbf{let}\ x = a\ \textbf{in}\ b \quad = \quad b\,[x/a]$	(80)
Elim-pair	$t \quad = \quad t[(x, y)/z, x/\pi_1 z, y/\pi_2 z]$	(81)
Def-cond	$(p \rightarrow f, g)\,x \quad = \quad \textbf{if}\ p\ x\ \textbf{then}\ f\ x\ \textbf{else}\ g\ x$	(82)
Def-guard	$p?\ a = \textbf{if}\ p\ a\ \textbf{then}\ i_1\ a\ \textbf{else}\ i_2\ a$	(83)
Def-ap	$ap(f, x) = f\ x$	(84)

Curry	$\overline{f}\ a\ b = f\ (a, b)$	(85)
Uncurry	$\widehat{f}\ (a, b) = f\ a\ b$	(86)
Composição monádica	$(f \bullet g)\ a = \mathbf{do}\ \{\, b \leftarrow g\ a; f\ b \,\}$	(87)
'Binding-μ'	$x \ggg= f \quad = \quad (\mu \cdot \mathsf{T}\, f)x$	(88)
Notação-do	$\mathbf{do}\ \{\, x \leftarrow a; b \,\} \quad = \quad a \ggg= (\lambda x \rightarrow b)$	(89)
'μ-binding'	$\mu\, x \quad = \quad x \ggg= id$	(90)
Sequenciação	$x \gg y \quad = \quad x \ggg= \underline{y}$	(91)