Leis do Cálculo Funcional (2022/23)

Funções

Natural-id	$f \cdot id = id \cdot f = f$	(1)
Assoc-comp	$(f \cdot g) \cdot h = f \cdot (g \cdot h)$	(2)
Natural-const	$\underline{k} \cdot f = \underline{k}$	(3)
Fusão-const	$f \cdot \underline{k} = \underline{f\ k}$	(4)
Leibniz	$\left\{ \begin{array}{l} f \cdot h = g \cdot h \\ h \cdot f = h \cdot g \end{array} \right. \Leftarrow f = g$	(5)

Produto

Universal-$\times$	$k = \langle f, g \rangle \Leftrightarrow \left\{ \begin{array}{l} \pi_1 \cdot k = f \\ \pi_2 \cdot k = g \end{array} \right.$	(6)
Cancelamento-$\times$	$\left\{ \begin{array}{l} \pi_1 \cdot \langle f, g \rangle = f \\ \pi_2 \cdot \langle f, g \rangle = g \end{array} \right.$	(7)
Reflexão-$\times$	$\langle \pi_1, \pi_2 \rangle = id_{A \times B}$	(8)
Fusão-$\times$	$\langle g, h \rangle \cdot f = \langle g \cdot f, h \cdot f \rangle$	(9)
Def-$\times$	$f \times g = \langle f \cdot \pi_1, g \cdot \pi_2 \rangle$	(10)
Absorção-$\times$	$(i \times j) \cdot \langle g, h \rangle = \langle i \cdot g, j \cdot h \rangle$	(11)
Natural-π_1	$\pi_1 \cdot (f \times g) = f \cdot \pi_1$	(12)
Natural-π_2	$\pi_2 \cdot (f \times g) = g \cdot \pi_2$	(13)
Functor-$\times$	$(g \cdot h) \times (i \cdot j) = (g \times i) \cdot (h \times j)$	(14)
Functor-id-$\times$	$id_A \times id_B = id_{A \times B}$	(15)
Eq-$\times$	$\langle f, g \rangle = \langle h, k \rangle \Leftrightarrow \left\{ \begin{array}{l} f = h \\ g = k \end{array} \right.$	(16)

Coproduto

Universal-$+$	$k = [f\ , g] \Leftrightarrow \left\{ \begin{array}{l} k \cdot i_1 = f \\ k \cdot i_2 = g \end{array} \right.$	(17)
Cancelamento-$+$	$\left\{ \begin{array}{l} [f\ , g] \cdot i_1 = f \\ [f\ , g] \cdot i_2 = g \end{array} \right.$	(18)
Reflexão-$+$	$[i_1\ , i_2] = id_{A+B}$	(19)
Fusão-$+$	$f \cdot [g\ , h] = [f \cdot g\ , f \cdot h]$	(20)
Def-$+$	$f + g = [i_1 \cdot f\ , i_2 \cdot g]$	(21)
Absorção-$+$	$[g\ , h] \cdot (i + j) = [g \cdot i\ , h \cdot j]$	(22)
Natural-i_1	$(i + j) \cdot i_1 = i_1 \cdot i$	(23)
Natural-i_2	$(i + j) \cdot i_2 = i_2 \cdot j$	(24)
Functor-$+$	$(g \cdot h) + (i \cdot j) = (g + i) \cdot (h + j)$	(25)
Functor-id-$+$	$id_A + id_B = id_{A+B}$	(26)
Eq-$+$	$[f\ , g] = [h\ , k] \Leftrightarrow \left\{ \begin{array}{l} f = h \\ g = k \end{array} \right.$	(27)

MISC. PRODUTO / COPRODUTO

Lei da troca $[\langle f, g\rangle \ , \langle h, k\rangle] \quad = \quad \langle [f \ , h], [g \ , k]\rangle$ (28)

CONDICIONAL

Natural-guarda $p? \cdot f \quad = \quad (f + f) \cdot (p \cdot f)?$ (29)

Def condicional de McCarthy $p \rightarrow f, g \quad = \quad [f \ , g] \cdot p?$ (30)

1.ª Lei de fusão do condicional $f \cdot (p \rightarrow g, h) \quad = \quad p \rightarrow f \cdot g, f \cdot h$ (31)

2.ª Lei de fusão do condicional $(p \rightarrow f, g) \cdot h \quad = \quad (p \cdot h) \rightarrow (f \cdot h), (g \cdot h)$ (32)

ISOMORFISMOS (α)

'Shunt-left' $h \cdot \alpha = k \quad \equiv \quad h = k \cdot \alpha^\circ$ (33)

'Shunt-right' $\alpha \cdot g = f \quad \equiv \quad g = \alpha^\circ \cdot f$ (34)

EXPONENCIAÇÃO

Universal-exp $k = \overline{f} \quad \Leftrightarrow \quad f = \mathsf{ap} \cdot (k \times id)$ (35)

Cancelamento-exp $f = \mathsf{ap} \cdot (\overline{f} \times id)$ (36)

Reflexão-exp $\overline{\mathsf{ap}} = id_{B^A}$ (37)

Fusão-exp $\overline{g \cdot (f \times id)} = \overline{g} \cdot f$ (38)

Def-exp $f^A = \overline{f \cdot \mathsf{ap}}$ (39)

Absorção-exp $f^A \cdot \overline{g} = \overline{f \cdot g}$ (40)

Natural-exp $g \cdot \mathsf{ap} = \mathsf{ap} \cdot (g^A \times id)$ (41)

Functor-exp $(g \cdot h)^A = g^A \cdot h^A$ (42)

Functor-id-exp $id^A = id$ (43)

FUNCTORES

Functor-F $\mathsf{F}(g \cdot h) \quad = \quad (\mathsf{F}\, g) \cdot (\mathsf{F}\, h)$ (44)

Functor-id-F $\mathsf{F}\, id_A \quad = \quad id_{(\mathsf{F}\, A)}$ (45)

INDUÇÃO

Universal-cata $k = (\!|\, g \,|\!) \quad \Leftrightarrow \quad k \cdot \mathsf{in} = g \cdot \mathsf{F}\, k$ (46)

Cancelamento-cata $(\!|\, g \,|\!) \cdot \mathsf{in} = g \cdot \mathsf{F}\, (\!|\, g \,|\!)$ (47)

Reflexão-cata $(\!|\, \mathsf{in} \,|\!) = id_{\mathsf{T}}$ (48)

Fusão-cata $f \cdot (\!|\, g \,|\!) = (\!|\, h \,|\!) \quad \Leftarrow \quad f \cdot g = h \cdot \mathsf{F}\, f$ (49)

Base-cata $\mathsf{F}\, f \quad = \quad \mathsf{B}\, (id, f)$ (50)

Def-map-cata $\mathsf{T}\, f \quad = \quad (\!|\mathsf{in} \cdot \mathsf{B}(f, id)|\!)$ (51)

Absorção-cata $(\!|g|\!) \cdot \mathsf{T}\, f \quad = \quad (\!|g \cdot \mathsf{B}(f, id)|\!)$ (52)

RECURSIVIDADE MÚTUA

Fokkinga	$\left\{\begin{array}{l} f \cdot in = h \cdot \mathsf{F}\,\langle f,g\rangle \\ g \cdot in = k \cdot \mathsf{F}\,\langle f,g\rangle \end{array}\right. \quad\equiv\quad \langle f,g\rangle = (\!|\langle h,k\rangle|\!)$	(53)
"Banana-split"	$\langle (\!|i|\!), (\!|j|\!)\rangle \quad=\quad (\!|(i\times j)\cdot\langle \mathsf{F}\,\pi_1, \mathsf{F}\,\pi_2\rangle|\!)$	(54)

COINDUÇÃO

Universal-ana	$k = [\!(g)\!] \quad\Leftrightarrow\quad \mathsf{out}\cdot k = (\mathsf{F}\,k)\cdot g$	(55)
Cancelamento-ana	$\mathsf{out}\cdot [\!(g)\!] = \mathsf{F}\,[\!(g)\!]\cdot g$	(56)
Reflexão-ana	$[\!(\mathsf{out})\!] = id_{\mathsf{T}}$	(57)
Fusão-ana	$[\!(g)\!]\cdot f = [\!(h)\!] \quad\Leftarrow\quad g\cdot f = (\mathsf{F}\,f)\cdot h$	(58)
Base-ana	$\mathsf{F}\,f \quad=\quad \mathsf{B}\,(id,f)$	(59)
Def-map-ana	$\mathsf{T}\,f \quad=\quad [\!(\mathsf{B}(f,id)\cdot\mathsf{out})\!]$	(60)
Absorção-ana	$\mathsf{T}\,f\cdot[\!(g)\!] \quad=\quad [\!(\mathsf{B}(f,id)\cdot g)\!]$	(61)

MÓNADAS

Multiplicação	$\mu\cdot\mu = \mu\cdot\mathsf{T}\,\mu$	(62)
Unidade	$\mu\cdot u = \mu\cdot\mathsf{T}\,u = id$	(63)
Natural-u	$u\cdot f \quad=\quad \mathsf{T}\,f\cdot u$	(64)
Natural-μ	$\mu\cdot\mathsf{T}\,(\mathsf{T}\,f) \quad=\quad \mathsf{T}\,f\cdot\mu$	(65)
Composição monádica	$f\bullet g \quad=\quad \mu\cdot\mathsf{T}\,f\cdot g$	(66)
Associatividade-$\bullet$	$f\bullet(g\bullet h) = (f\bullet g)\bullet h$	(67)
Identidade-$\bullet$	$u\bullet f = f = f\bullet u$	(68)
Associatividade-$\bullet/\cdot$	$(f\bullet g)\cdot h \quad=\quad f\bullet(g\cdot h)$	(69)
Associatividade-$\cdot/\bullet$	$(f\cdot g)\bullet h \quad=\quad f\bullet(\mathsf{T}\,g\cdot h)$	(70)
μ versus $\bullet$	$id\bullet id \quad=\quad \mu$	(71)

DEFINIÇÕES *ao ponto* ('POINTWISE')

Igualdade extensional	$f = g \quad\Leftrightarrow\quad \langle\forall\, x\ ::\ f\ x = g\ x\rangle$	(72)
Def-comp	$(f\cdot g)\ x \quad=\quad f\ (g\ x)$	(73)
Def-id	$id\ x \quad=\quad x$	(74)
Def-const	$\underline{k}\ x \quad=\quad k$	(75)
Notação-λ	$f\ a = b \quad\equiv\quad f = \lambda a \rightarrow b$	(76)
Def-split	$\langle f,g\rangle\ x \quad=\quad (f\,x, g\,x)$	(77)
Def-$\times$	$(f\times g)\,(a,b) \quad=\quad (f\,a, g\,b)$	(78)
Def-cond	$(p\rightarrow f,g)\,x \quad=\quad \textbf{if}\ p\ x\ \textbf{then}\ f\ x\ \textbf{else}\ g\ x$	(79)
Def-proj	$\pi_1\,(x,y) \ =\ x \qquad\wedge\qquad \pi_2\,(x,y)\ =\ y$	(80)
Elim-let	$\textbf{let}\ x = a\ \textbf{in}\ b \quad=\quad b\,[x/a]$	(81)
Elim-pair	$t \quad=\quad t[(x,y)/z, x/\pi_1 z, y/\pi_2 z]$	(82)
Def-ap	$ap(f,x) = f\ x$	(83)
Curry	$\overline{f}\ a\ b = f\ (a,b)$	(84)
Uncurry	$\widehat{f}\ (a,b) = f\ a\ b$	(85)

Composição monádica	$(f \bullet g)\ a = \mathbf{do}\ \{\, b \leftarrow g\ a; f\ b \,\}$	(86)
'Binding as μ'	$x \gg\!= f \quad = \quad (\mu \cdot \mathsf{T}\, f) x$	(87)
Notação-do	$\mathbf{do}\ \{x \leftarrow a; b\} \quad = \quad a \gg\!= (\lambda x \rightarrow b)$	(88)
'μ as binding'	$\mu\, x \quad = \quad x \gg\!= id$	(89)
Sequenciação	$x \gg y \quad = \quad x \gg\!= \underline{y}$	(90)